Binomial Distribution

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

This is the probability density function of the Binomial distribution. $P(X = x)$ tells us we are dealing with a probability when x is a certain value. p is the probability of success of an event. n is the total number of trials. x is the number of successes. The construction $\binom{n}{x}$ means the number of ways x objects can be chosen from a total n objects. If you have, say, 20 cars and all have a probability of selling of 0.80, and you want to determine the probability of 10 being sold – you have no idea which 10 might be sold, so you have to take account for all of the possible combinations out of the 20 total cars.

The other two distributions that we have covered are the Poisson and Normal distributions. The former is for random events with a very small probability of happening (e.g. *"Telephone calls arrive at a call centre at the rate of 50 per hour. Find the probabilities of 0, 1 or 2 calls arriving in any 6 minute period."* would be a typical Poisson question, the latter is for continuous[1] distributions (e.g. *"An industrial process mass produces an item whose weights are normally distributed with mean 18.5kg and standard deviation[2] 1.5kg. What is the probability that an item chosen at random weighs 21.5kg?"* would be a typical Normal question).

There are four requirements that must be followed for the use of the binomial distribution:

1. There are a fixed number of trials, n
2. There are two possible outcomes, success or failure
3. There is a fixed probability of success, p
4. Trials are independent of each other

For example, seeds have a probability of germinating of 0.9. We need to find the probability of five seeds germinating, given that six seeds are sown.

$$P(X = 5) = \binom{6}{5} 0.9^5 (1 - 0.9)^{6-5}$$
$$P(X = 5) = 6 \times 0.59049 \times 0.1^1$$
$$P(X = 5) = 0.354294$$

[1] relating to items that can have any value within a given range (e.g. height of a person can be 1.66m, 2.0956m, or any other value to as many or as few decimal places) the opposite of this is something that is discrete, and can only take specific values (e.g. number of sweets in a jar, can only be 0,1,2 etc. – you cannot have 1.66 sweets or 2.0956 sweets)

[2] this is a measure of the average difference of a value from the mean. In an exam this will probably be a small single-digit value.

If we wanted to find out the probability of six out of the six seeds germinating, instead of repeating to format of the above equation, we can use the binomial recurrence formula

$$P(X = x + 1) = \frac{n - x}{x + 1} \times \frac{p}{(1 - p)} \times P(X = x)$$

So

$$P(X = 6) = \frac{6 - 5}{6} \times \frac{0.9}{(1 - 0.9)} \times P(X = 5)$$
$$P(X = 6) = \frac{1}{6} \times 9 \times 0.354294$$
$$P(X = 6) = 0.531441$$

Note
If we had started by calculating $P(X = 0)$, we could have found the probability for all x-values.
$P(X = 0)$ is inputted to get $P(X = 1)$, $P(X = 1)$ is inputted to get $P(X = 2)$ and so on.

See also
- Normal Distribution
- Poisson Distribution
- Combinations

References
Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Statistics and Mechanics - Year 1*. London: Pearson Education. pp.88-89.
Graham, D., Graham, C. and Whitcombe, A. (1986). *A-level Mathematics*. London: Charles Letts. p.150.